MEMS Resonator Parameter Estimation from Fast Frequency Sweeps

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***Abstract*— In this paper, we introduce a new approach to the estimation of the parameters of high quality factor, low natural frequency resonators. Our approach is based on transient measurements of the slowly-varying amplitude and phase response of a resonator to an arbitrary nearly-periodic drive stimulus. We show how this approach may be employed for estimating the parameters of nonlinear resonators, with a parameter estimation procedure that does *not* rely on a closed-form expression of the transient response, nor on an iterative algorithm (it results in a *linear* least squares problem), is simple to implement and is very amenable to integration. We illustrate our approach with simulations of fast frequency sweeps, highlight its benefits compared to steady-state response fitting, and study its robustness to measurement noise.**

Keywords—resonators; parameter estimation; slowly-varying amplitude and phase approximation.

I. Introduction

Several applications require that a resonator's system-level parameters –such as quality factor Q or natural frequency f_0– be assessed through electrical measurements. This presents interesting challenges for MEMS resonators, used for resonant sensing or for kinetic energy harvesting for instance, as these devices typically have low natural frequencies (on the order of a few kHz for inertial resonant sensing, a few Hz for human body motion energy harvesting) and large quality factors ($10^4 - 10^6$ for resonant sensors, $10^2 - 10^3$ for harvesters). Consequently, the experimental determination of the parameters of such resonators requires a significant amount of time, limited by what is deemed practical in terms of response time and environmental drift.

For example, parameter estimation methods relying on the measurement of N points of the steady-state response of the resonator (e.g. [1-2]) require a time that is significantly larger than NQ/f_0, during which the environmental conditions must be precisely monitored, and controlled or compensated for. Alternatively, some methods seek to exploit transient measurements to perform parameter estimation with a more reasonable time scale; the golden standard for high Q, low f_0 resonators is the "ringdown" method, which consists in measuring the decaying oscillations of a free resonator. Several variants of this method have been proposed in recent years, extending its use from the case of a single degree-of-freedom resonator to multiple degrees-of-freedom, nonlinear resonators [3-5]. Although ringdown methods are the fastest and are intrinsically immune to feedthrough, they also have a few limitations: some parameters cannot be identified (e.g. transduction gain), and their implementation may be difficult, especially for the case of nonlinear ringdown, as proposed in [4]. This method relies on the repeated evaluation – inside of a nonlinear least-squares fitting procedure – of a closed-form analytical solution of the equation governing the slowly-varying amplitude and phase of the decaying resonator. While this is hardly an issue when the method is run on a PC, its integration in a system-on-chip is more difficult. Furthermore, its convergence is not guaranteed. Finally, depending on the nonlinearity, there may be no closed-form expression for the slowly-varying amplitude and phase of the resonator. In that case, one may then fit numerical simulation results to the experimental data, but the repeated calls to a numerical simulation tool inside of the nonlinear least squares procedure would also likely have its drawbacks.

In this paper, we introduce a new approach to high Q, low f_0 resonator parameter estimation based on transient measurements of the slowly-varying amplitude and phase response of a resonator to an arbitrary nearly-periodic drive stimulus. We show how this approach may be employed for estimating the parameters of nonlinear resonators, with a parameter estimation procedure that does *not* rely on a closed-form expression of the transient response, nor on an iterative algorithm (it results in a *linear* least squares problem), is simple to implement and is very amenable to integration.

In section II, we describe the fundamentals of our approach, in the case of constant amplitude, fast frequency sweeps. We highlight the key points that may impact parameter estimation accuracy. In section III we illustrate our approach with simulations and study its robustness to measurement noise. Finally, section IV contains some conclusions and perspectives.

II. Proposed approach

For the sake of simplicity and of outlining the key points of our approach, we present it in a restrained framework. We consider here a high Q, low f_0, single degree-of-freedom resonator, with polynomial restoring and damping forces, subject to a quasi-harmonic driving force whose instantaneous frequency is close to f_0. We suppose the measured signal is free from feedthrough (this hypothesis is discussed further on), so that it does represent only the position of the resonator over time. This framework can be summed up with the following set of equations:

$$v_d(t) = V_d(t) \times \sin(\int \omega_d(t)dt) \quad (1)$$

$$\omega_0^2 \times (1 + \sum_{i=1}^{n} \gamma_i x^{2i})x +$$

$$\frac{\omega_0}{Q} \times (1 + \sum_{i=1}^{n} \alpha_i x^{2i})\dot{x} + \ddot{x} = G_d \times v_d(t) \quad (2)$$

$$v_m(t) = G_m \times x(t) \quad (3)$$

where $v_d(t)$ is the drive voltage with slowly varying amplitude $V_d(t)$ and angular frequency $\omega_d(t)$ – $V_d(t)$ and $\omega_d(t)$ must be adequately chosen by the user – where $\omega_0 = 2\pi f_0$, $x(t)$ is the position of the resonator, the γ_i and α_i coefficients respectively represent the restoring and damping force nonlinearity, G_d and G_m are transduction/readout gains, and $v_m(t)$ is the measured voltage at the output of the resonator readout. The latter signal is typically measured with a lock-in amplifier which delivers a complex, slowly-varying signal $Z(t)$ whose real and imaginary part respectively correspond to the components of $v_m(t)$ in phase and in quadrature with $v_d(t)$. From (1-3), one may use harmonic balance to show that $Z(t)$ is the solution of

$$2j\omega_d(t)\dot{Z} + (C_0 + \sum_{i=1}^{n} C_i |Z|^{2i} - \omega_d(t)^2)Z = gV_d(t) \quad (4)$$

where the real parameter g, and the complex parameters $C_0, \ldots, C_1$ are trivially related to the unknown parameters in (1-3), e.g.

$$g = G_d / G_m \quad (5)$$

$$C_0 = \omega_0^2 \times (1 + \frac{j}{Q}) \quad (6)$$

$$C_1 = \frac{\omega_0^2}{4} \times (3\gamma_1 + j\frac{\alpha_1}{Q}) \quad (7)$$

Note that the derivation of (4-7) from (1-3) relies on several simplifying assumptions: neglecting higher-order derivatives $\ddot{Z}$ and $\dot{\omega}_d$, approximating $\omega_d \approx \omega_0$ in some terms, etc. However these have little consequence on our approach, which may be used, with the same ease and to the same effect, with or without resorting to them.

Let us now define $\boldsymbol{\theta} = (g, C_0, \ldots, C_n)$ the set of unknown parameters, and synthetically rewrite (4) as

$$F\big(t, Z, \dot{Z}, \boldsymbol{\theta}\big) = 0 \quad (8)$$

Now, it is important to notice that, although (4) is a nonlinear ordinary differential equation in terms of $Z(t)$, it is linear with respect to the parameters one seeks to estimate. This means that the unknown parameters $\boldsymbol{\theta}$ can be estimated as the unique solution of the following *linear* least squares problem

$$\widehat{\boldsymbol{\theta}} = \arg\min \sum_{k=1}^{N} \left(F\big(t_k, Z_k, \dot{Z}_k, \boldsymbol{\theta}\big)\right)^2 \quad (9)$$

where the t_k are the N instants at which the output of the lock-in amplifier $Z(t)$ is sampled, $Z_k = Z(t_k)$ and $\dot{Z}_k = \dot{Z}(t_k)$. A practical difficulty arises from the fact that typical lock-in amplifiers have no $\dot{Z}(t)$ output. The idea here is then simply to estimate $\dot{Z}_k$ from the outputs Z_k, for example, if $Z(t)$ is uniformly sampled, by using the central finite difference scheme

$$\hat{\dot{Z}}_k = \frac{Z_{k+1} - Z_{k-1}}{t_{k+1} - t_{k-1}} \quad (10)$$

and to estimate the unknown parameters as

$$\widehat{\boldsymbol{\theta}} = \arg\min \sum_{k=2}^{N-1} \left(F\left(t_k, Z_k, \hat{\dot{Z}}_k, \boldsymbol{\theta}\right)\right)^2 \quad (11)$$

which is still a *linear* least squares problem, with a unique solution. Furthermore, the computation of $\widehat{\boldsymbol{\theta}}$ as the solution of the normal equations only requires straightforward calculations which can be integrated with ease on a digital platform.

Note that the same "trick" could be used, in principle, to estimate the unknown parameters directly from samples of $v_m(t)$, without making any restrictive assumption as to the nature of $v_d(t)$, as (2-3) are also nonlinear ordinary differential equations that are linear with respect to the parameters of interest. However, for the same accuracy, this would require many more samples of $v_m(t)$ than we need of $Z(t)$, and the estimation of the derivatives $\dot{x}(t)$ and $\ddot{x}(t)$ would likely be less precise than that of $\dot{Z}(t)$ (it would not benefit from the noise rejection provided by the lock-in detection technique). In fact, the accuracy with which $\dot{Z}(t)$ can be estimated is a key element of the method, as explained and illustrated in section III.

Another key element of the method is that the relation between the position of the resonator $x(t)$ and the measured voltage $v_m(t)$ (3) must be perfectly known, up to a multiplicative coefficient. This is also in part what allows us to formulate the parameter estimation problem as a linear least squares problem. For instance, if there is an unknown amount of feedthrough in $v_m(t)$ so that

$$v_m(t) = G_m \times x(t) + G_{ft} \times v_d(t) \quad (12)$$

the parameter estimation problem (where the parameters now include G_{ft}) becomes a nonlinear least squares problem, which must be solved iteratively. It should be noted that there exists in the literature several techniques to minimize, compensate for or cancel out feedthrough altogether (e.g. [6-7]).

Finally, it is also interesting to note that the proposed approach degenerates to a nonlinear ringdown technique if $v_d(t) = 0$. However, as opposed to the technique proposed in [4], which relies on a closed form expression for $Z(t)$ and leads to a nonlinear optimization problem, our approach relies on approximating $\dot{Z}(t)$ and leads to a linear optimization problem.

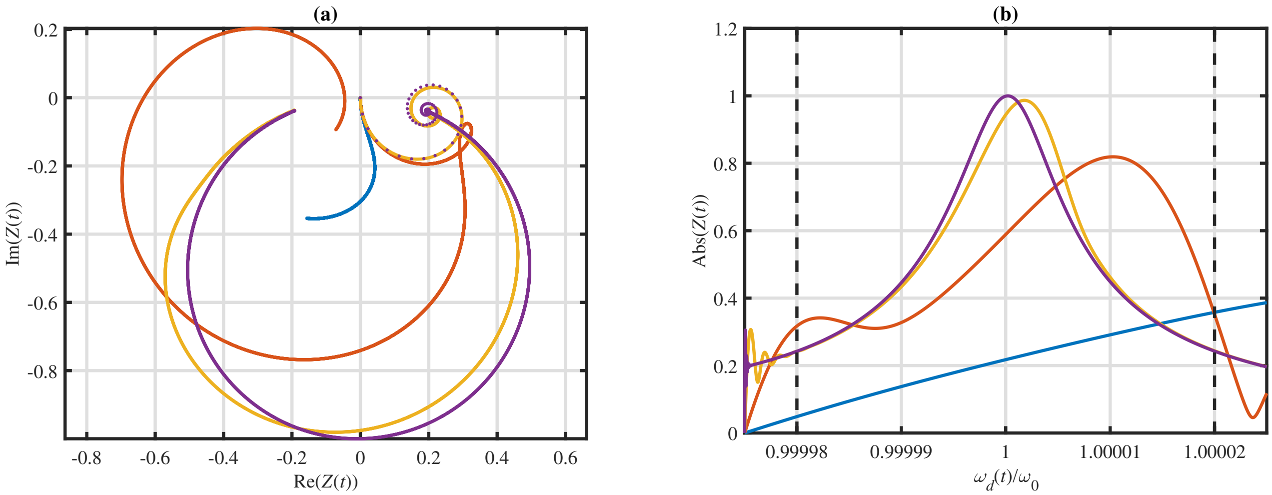

Fig. 1. Simulated Nyquist plot of linear resonator response (a) and amplitude of resonator response (b), for $T_{sweep} = [1,10,10^2,10^3] \times Q/\omega_0$ (resp. blue, orange, yellow, purple lines). The dashed vertical lines delimit the points over which fitting is performed.

III. Results

In this section, we report numerical simulation results obtained with our approach using fast linear frequency sweeps of fixed amplitude ($V_d(t) = V_0$) with duration T_{sweep} over a bandwidth $\Delta\omega$ centered around ω_0

$$\omega_d(t) = \omega_0 + \Delta\omega \times (t - \frac{T_{sweep}}{2})) \tag{13}$$

Fig. 1 represents typical results obtained by simulating the evolution of $Z(t)$ from (4) with $\Delta\omega/\omega_0 = 10/Q$ and varying values of T_{sweep} ranging from Q/ω_0 to $1000 \times Q/\omega_0$, for a resonator with $\omega_0 = 1\,\mathrm{rad.s^{-1}}$ and $Q = 10^5$. The gain parameter g is set to 1. The simulation is performed using Gear's 2$^{\mathrm{nd}}$ order formula, which, in the linear resonator case, can be transformed into an unconditionally stable explicit scheme. The fixed timestep of the simulation is $T_{sweep}/10^4$. The use of this non-iterative, unconditionally-stable time integration scheme provides excellent guarantees as to the smallness of the error between the simulated response and the actual response.

Since the amplitude of the swept frequency response is significant provided $T_{sweep} \geq Q/\omega_0$, one may expect that accurate information about the system parameters can be recovered even for moderate sweep durations. However, for short sweep durations, the shape of the response, whether in terms of amplitude or in the complex plane, is heavily distorted compared to the Lorentzian or the circle on may expect from the steady-state response. One may thus expect significant errors between the actual parameters and those obtained by fitting a steady-state model to the transient response.

To verify these two points, we calculate the values of ω_0 and Q estimated by fitting a transient model, as per our approach (10-11), to the simulated points – or, more precisely, to the simulated points decimated with a 1:10 ratio, leaving out the first and last 10% of the simulation: this lets us retain the most significant points to do the fitting, while the 1:10 decimation makes sure that the approximation error of the finite difference formula (11), whose influence we want to assess, is on the order of two orders of magnitude larger than that of our numerical solver. We can also compare these estimates to those obtained by fitting a steady-state model to these same points, i.e. setting

$$\hat{\dot{Z}}_k = 0 \tag{14}$$

in (10). We represent the errors made with both methods in Fig. 2. Regardless of the method, the error on g is nearly identical to the error on Q, so it is not represented here. As may be expected, both methods give good results for long sweep durations, but the results obtained with our approach are always several orders of magnitude better than with the steady-state model, although the eye cannot detect any significant difference between the transient response at $T_{sweep} = 1000 \times Q/\omega_0$ and the steady-state response of a linear resonator. This highlights that accounting for transient terms is extremely relevant for precise parameter estimation, even for sweep durations that are large with respect to the response time of the resonator. Furthermore, the proposed approach gives excellent results, even with very short sweep durations, at least in the idealized, noise-free case (in which the error is mostly due to the approximation of $\dot{Z}_k$ by $\hat{\dot{Z}}_k$). Note that, in Fig.2, the error on the estimation of parameter ω_0 is given reslatively to ω_0/Q.

When either process or measurement noise is present (i.e. when there is an additive random term appearing in (2) or (3)), the precision at low sweep durations is compromised. For example, in the case of measurement noise, only a noisy version of $Z(t)$ is available to perform the fitting. Denoting this by $Z_n(t)$ we have

$$Z_{n,k} = Z_k + n_k \tag{15}$$

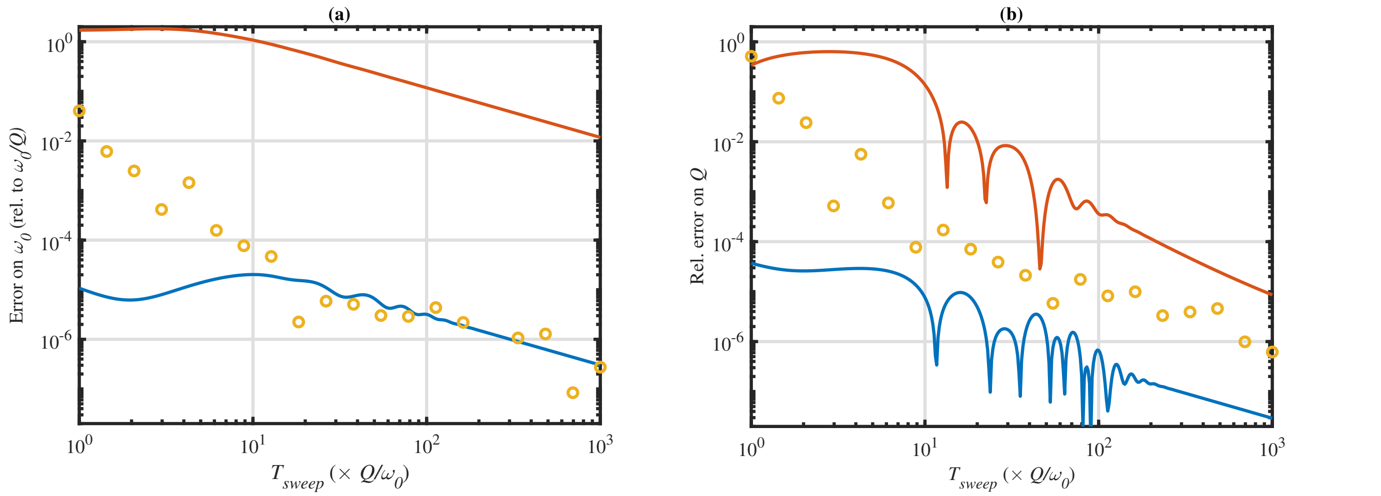

Fig. 2. Relative error on ω_0 (a) and on Q (b) obtained with our approach (blue lines) and by fitting a steady-state model (orange lines) to the simulation results. The circles represent these errors when accounting for measurement noise, averaged over 10^5 simulations.

where n_k is the additive measurement noise at time t_k. Applying a central finite difference scheme to $Z_n(t)$ in order to estimate $\dot{Z}(t)$ yields

$$\frac{Z_{n,k+1}-Z_{n,k-1}}{t_{k+1}-t_{k-1}} = \hat{\dot{Z}}_k + \frac{n_{k+1}-n_{k-1}}{t_{k+1}-t_{k-1}} \tag{16}$$

meaning that there is an error term in the finite difference formula whose value is all the greater as the noise is large, and as the time-step (and T_{sweep}) is small. Furthermore, for a fixed number of samples, the variance of n_k is typically inversely proportional to T_{sweep}. To illustrate this, we also represent in Fig. 2, the results obtained in a case where white measurement noise is present (averaged over 10^5 simulations). The results with process noise are not significantly different. This illustrates more realistically the trade-off between the sweep duration and the precision of the estimation. In spite of the degradation at short T_{sweep} values, the parameters are better estimated with our approach in the noisy case than with the steady-state model in the noise-free case. Sample noisy responses are plotted in Fig. 3 for reference.

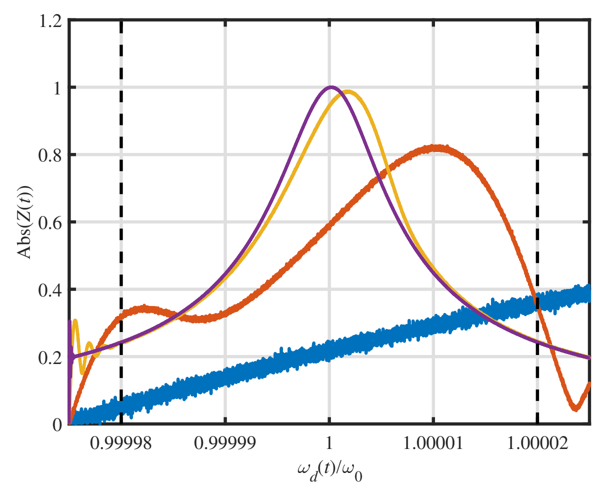

Fig. 3. Simulated amplitude of resonator response, for $T_{sweep} = [1, 10, 10^2, 10^3] \times Q/\omega_0$ (resp. blue, orange, yellow, purple lines) in the presence of measurement noise (the same amount as for deriving the results of Fig. 2).

IV. Conclusion

In this paper, we have outlined the main advantages and the few drawbacks of a new approach to parameter estimation for high Q, low f_0 resonators. We have shown our approach has many of the advantages of nonlinear ringdown – it may even reduce to the same experimental protocol – with the further benefit that the parameter estimation problem reduces to a linear least squares problem, whose solution is unique, and whose practical implementation or integration in a system-on-chip is highly tractable.

Furthermore, simulations have shown that a simple installment of our approach, based on frequency sweeps, yields significant improvements over steady-state methods, even in the presence of noise.

These simulations also illustrated the importance of accurately estimating $\dot{Z}(t)$ to get the best out of frequency sweep data, even with slow frequency sweeps. Note that, in the lab, the practical implementation of fast frequency sweeps is simple enough, with a lock-in amplifier (this is illustrated in our poster with ONERA's VIG gyroscope), or with an oscilloscope capable of down-converting the resonator signal to its phase and quadrature components and an arbitrary waveform generator, for example.

Our current work is dedicated to the implementation of this approach in a system-on-chip, using various feedthrough cancellation techniques (active, or with subharmonic pulsed drive [7]), to the characterization of coupled resonators (e.g. through simultaneous multitone sweeps), and to the optimization of $V_d(t)$ and $\omega_d(t)$, to achieve the best compromise between power consumption and precision.

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